UNIT-1

CO1: To understand the basic concept of mode of heat transfer.

CO-3: To analyze the complex problems of heat transfer with proper boundary conditions.

SN	CONTENTS	HOURS
1	Introduction: Objective, Scope and outcome of the course	1
2	Heat transfer processes, conduction and radiation. Fourier's law of heat conduction, thermal conductivity, thermal conductivity of solids, liquids and gases, effect of temperature on thermal conductivity. Newton's law of cooling, definition of overall heat transfer coefficient. General parameters influence the value of heat transfer coefficient	4
	Conduction: General 3-Dimensoinal conduction equation in Cartesian, cylindrical and spherical coordinates; different kinds of boundary conditions; nature of differential equations; one dimensional Heat conduction with and without heat generation; electrical analogy; heat conduction through composite walls; critical thickness of insulation.	3

Heat transfer and Thermodynamics

Thermodynamics :

- Thermodynamics" deals with the amount of energy in form of heat or work during a process and only considers the end states in equilibrium.
- How much heat is transferred (dQ)
- How much work is done (dW)

Heat transfer

- Heat Transfer" deals with the ***rate of heat transfer*** thus, Heat transfer deals with ***time*** and non equilibrium phenomena. Heat can only transfer when there is ***a temperature gradient*** exists in a body and which is indication of non equilibrium phenomena.
- How (with what **modes**) dQ is transferred
- At what **rate** dQ is transferred
- Temperature distribution inside the body

Modes of Heat transfer

Conduction :

- The process by which heat directly transmitted through the material of a substance when there is a difference of temperature between adjoining regions, without movement of the material

<u>Fourier's law of conduction:</u> IT states that the heat transferred through conduction is proportional to

- Area of cross section perpendicular to the direction of heat flow. (A)
- Temperature difference between the points causing the heat flow. (dT/dx)
- Inversely proportional to the thickness of the material along which heat is flowing.(x)

$$q = kA\, dT/dx$$

Where,

- q = heat transfer (W)
- k = Thermal conductivity (W/mK)
- dT/dx = Temperature gradient (K)

Convection :

- An energy transfer across a system boundary due to a temperature difference by the combined mechanisms of intermolecular interactions and bulk transport. Convection needs fluid matter.

<u>Newton's law of cooling:</u>

- Rate of convective heat transfer is directly proportional to heat transfer surface area, convective heat transfer coefficient and temperature difference.

a) natural convection b) forced convection

$$Q = hA\, dT$$

Q = Convection heat transfer (W)

h = Heat transfer coefficient (W/m^2K)

dT = Temperature gradient (K)

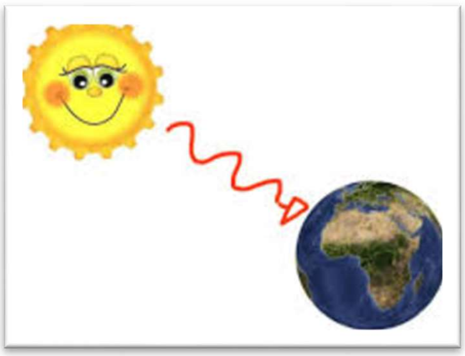
<u>Radiation</u> :

- Radiation is the emission or transmission of energy in the form of waves or particles through space or through a material medium. This includes: electromagnetic radiation, such as radio waves, microwaves, infrared, visible light, ultraviolet, x-rays, and gamma radiation (γ)

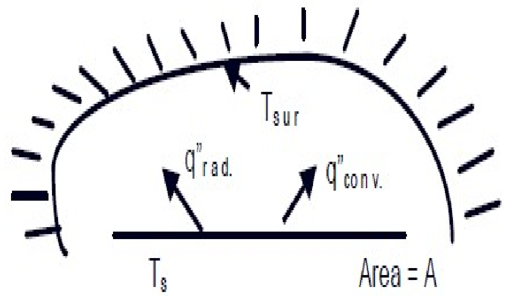
Stefan boltzmann law :

- It state that the total radiant heat power emitted from a surface is proportional to the fourth power of its absolute temperature.

  However, the rate of radiation heat exchange between a small surface and a large surrounding is given by the following expression



Radiation from sun



Radiation heat transfer

$$Q = \varepsilon.\sigma.A.(T_s^4 - T_{surr}^4)$$

Q = Radiation heat transfer (W)

σ = Boltzmann constant (W/ m^{2}K^4)

Ɛ = Emissivity

Thermal Conductivity

When a system gains heat, it stores some of its heat energy and transports the remaining heat energy to some other system. The ability of a system to transport heat energy is referred to as Thermal conductivity of the system. Basically, it is a Transport property of a system. Thermal conductivity is represented by k. The unit of thermal conductivity as we have seen earlier is W/m*K.

Thermal conductivity in solids, liquids and gases

What governs conduction in solids, liquids and gases?

Before analyzing thermal conductivity for different phases, let us look at the phenomena which govern heat conduction through solids, liquids, and gases.

In solids, heat can be conducted through two mechanisms. First is lattice vibrations and the second is Flow of free electrons. Increased lattice vibrations facilitate the transport heat energy through the medium. The flow of free electrons increases electrical conductivity. This also helps in the process of diffusion of heat energy through the medium.

In liquids and gases, heat conduction occurs mainly through two mechanisms. First is the collision between atoms, molecules or ions, and second is molecular diffusion. As the number of collisions increases, the exchange of energy among molecules increases. This helps in the transport of heat energy through the medium. Molecular diffusion is the random movement of molecules in a medium. As the random movement of molecules increases, it obstructs the transport of heat energy in a particular direction.

On what factors does the thermal conductivity depend for metals, non-metals and alloys?

As we have seen above, the conduction of heat through solids is dependent on two effects, namely lattice vibrations and flow of free electrons. The thermal conductivity is obtained by adding lattice and electronic components.

In Pure metals, the electronic effect plays a dominant role. Thus, they have relatively higher values of thermal conductivity. For Pure metals, $k \sim ke$.

In Non-metals, the lattice vibrations effect plays a dominant role. Non-metals generally have high electric resistance, which obstructs the flow of electrons. Therefore, for non-metals k ~ kl.

The lattice component of thermal conductivity strongly depends on the way the molecules are arranged. For example, wood, which is an amorphous solid (molecules are arranged in highly disorderly manner), has relatively lower values of thermal conductivity and act as a thermal insulator. Now consider diamond. It is a highly ordered crystalline solid. Thus it has the highest thermal conductivity at room temperature. Beryllium Oxide (BeO), also a non-metal, has relatively higher thermal conductivity due to its crystallinity.

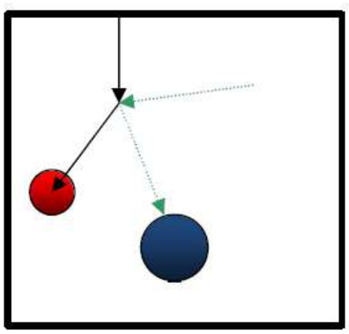
Metals are good electrical and heat conductors because they have free electrons as well as lattice vibrations. On the other hand, non-metals do not have free electrons, meaning they are electrically non-conducting materials. And in general non-metals like wood are thermally non-conducting materials. However, non-metals like diamond and Beryllium Oxide are good heat conductors. As a result, such materials find widespread use in the electronics industry.

E.g. diamond heat sinks used for cooling electronic components.

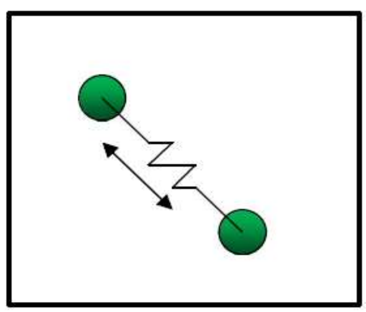
Pure alloys have high thermal conductivity. One would expect an alloy made of two metals of thermal conductivity k_1 and k_2 to have a conductivity k between k1 and k2. Surprisingly, this is not the case. The thermal conductivity of an alloy of two metals is usually much lower than that. For example, the thermal conductivities of Copper and Aluminum are 401 W/m°C and 237 W/m°C respectively.

Material	Thermal Conductivity, W/m K
Copper	401
Silver	429
Gold	317
Aluminum	237
Steel	60.5
Limestone	2.15
Bakelite	1.4
Water	0.613
Air	0.0263

Let us try to gain an insight into the basic concept of thermal conductivity for various materials. The fundamental concept comes from the molecular or atomic scale activities. Molecules/atoms of various materials gain energy through different mechanisms. Gases, in which molecules are free to move with a mean free path sufficiently large compared to their diameters, possess energy in the form of kinetic energy of the molecules. Energy is gained or lost through collisions/interactions of gas molecules.



Kinetic energy transfer between gas molecules.



Lattice vibration may be transferred between molecules as nuclei attract/repel each other.

Solids, on the other hand, have atoms/molecules which are more closely packed which cannot move as freely as in gases. Hence, they cannot effectively transfer energy through these same mechanisms. Instead, solids may exhibit energy through vibration or rotation of the nucleus. Hence the energy transfer is typically through lattice vibrations.

Another important mechanism in which materials maintain energy is by shifting electrons into higher orbital rings. In the case of electrical conductors the electrons are weakly bonded to the molecule and can drift from one molecule to another, transporting their energy in the process. Hence, flow of electrons, which is commonly observed in metals, is an effective transport mechanism, resulting in a correlation that materials which are excellent electrical conductors are usually excellent thermal conductors.

UNIT - I

Thermodynamics and Heat Transfer

• Energy will be exist in various form. In this Subject we are primarily interested in Heat energy, which is the form of energy that can be transferred from one system to another as a result of temperature difference. The science deals with the determination of rates of such energy transfer is heat transfer.

Thermodynamics is concerned with the amount of heat transfer as a system undergoes a process from one equilibrium to another and it gives no indication about how long the process will take. The thermodynamic analysis simply tell us how much heat must be transfered to realise a specified change of state to satisfy the conservation of energy principle.

The basic requirment of Heat transfer is presence of a temperature difference. It is the driving force for the ~~for~~ Heat transfer The rate of heat transfer is certain direction depends on the magnitude of the temp. gradient. (The temp. difference per unit length or rate of change of temp.) in that direction.

Application areas of Heat transfer

*) Mechanical Engg. : In Boilers, heat exchangers turbine system, internal combustion engine

*) mettalurgical Engg :- In Furnace, heat treatment plant. of component

*) electrical Engg : cooling system for electric motors, generators, transformers.

*) chemical Engg: In process equipment used in refineries, chemical plant. etc

*) Nuclear energy : design of nuclear fuel rods against possible burnout etc.

*) Aerospace Engg. In design of aircraft system rockets. missile. etc.

*) Cryogenic Engg. : In production, storage & transportation of utilization of cryogenic liquid for various industrial, research & defence application

*) Civil Engg :- In design of suspension bridges, railway track, air conditioning & insulation of buildings.

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modes of heat transfer

1) conduction

Conduction occurs mostly in a stationary mediums. It is mode of heat transfer in which energy exchange takes place from region of high temp to that of low temp by direct molecular interaction and by the drift of electrons

The thermal energy in solid may be conducted by two mechanism, migration of free electrons and lattice vibration. These two effects are additives, but in general the transport due to free electrons is more effective than transport due to vibrational energy in lattice structure.

In non-metals the energy transfer is due to lattice vibration only

The conduction heat transfer in liquid and gases occurs due to collision & diffusion of molecules during their random motion.

In gases. The mechanism of heat conduction is simple. The kinetic energy of molecule is the function of temp

PPB

These molecules are in continuous random motion exchanging energy and momentum when a molecule from the high temp region collides with molecule from the low temp region, it losses energy by collision.

In liquids the Mechanism of heat is near to that gases, However, the molecules are more closely spaced & intermolecular forces come into play.

② Convection

The convection is mode of heat transfer in which the energy is transported by moving fluid particles. The convection heat transfer comprises two Mechanism.

i) Transfer of energy due to random motion (diffusion)

ii) Transfer of energy by bulk or macroscopic motion of fluid (advection)

In absence of of any bulk fluid motion, the heat transfer occurs by pure conductions. The convection heat transfer due to superposition of energy transfer by random motion of molecules and by bulk motion of fluid

∴ The convection is not Fundamentally different mode of heat transfer. It consist of

" conduction from surface to adjacent layer of fluid + energy transfer due to mass transfer + conduction to adjacent layer of fluid to receiving surface ".

It classified into two type that is based on nature of fluid flow

a) Forced convection : if fluid motion is artificially induced by pump, fan, or blower, that forces the fluid over surface to flow. the heat transfer is said to be forced.

b) Natural convection : if fluid motion is setup by buoyancy effects. resulting from density difference caused by temperature difference in the fluid, the heat transfer by natural convection.

③ Radiation:

Thermal radiation is the energy emitted by substance because of its temp. The radiation energy emitted by a body is transmitted in the space in the form of electromagnetic waves according maxwell wave theory.

Transfer of energy require the presence of material medium. radiation does not. in fact radiation heat transfer is more efficient in Vacume. Thermal radiation occurs in the region of wavelength 0.1 μm to 100 μm on electromagnetic spectrum.

Laws of Heat transfer

Their are different fundamental and Subsidiary law of Heat transfer

1) Fourier law of heat conduction

whenever, a temperature gradient exist in a body, there is an energy transfer from the high temp region to low temp region. The fourier law state that "rate of heat conduction per unit area (heat flux) is directly proportional to temp gradient

$$\frac{Q}{A} \propto \frac{dT}{dx}$$

$$\therefore \quad q = \frac{Q}{A} = -K\frac{dT}{dx} \quad \therefore \quad Q = -KA\frac{dT}{dx}$$

- q - heat flux (W/m^2)
- Q - rate of heat transfer (W)
- A - heat transfer area in m^2 normal to direction of heat flow.

$\frac{dT}{dx}$ = Temp. gradient in °c/m

k - constant of proportionality called thermal conductivity ($W/m°c$ or $W/m·K$)

-ve sign inserted to make natural flow +positive heat quantity)

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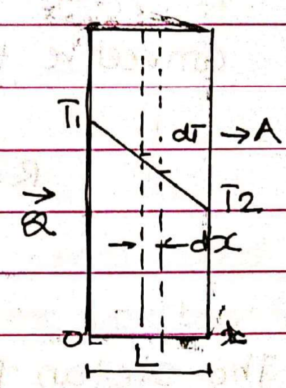
~~Newton's law of cooling~~

A simple case of one dimensional steady state heat flow through a plane wall is shown in fig for constant thermal conductivity. K. & heat transfer area A



$$q = \frac{Q}{A} = -K\frac{dT}{dx}$$

integrating above eq.

$$\frac{Q}{A}\int_0^L dx = -K\int_{T_1}^{T_2} dT$$

$$\frac{Q}{A} L = -K(T_2 - T_1)$$

$$\frac{Q}{A} L = K(T_1 - T_2) \quad \therefore Q = KA\frac{(T_1 - T_2)}{L}$$

$$\therefore \frac{dT}{dx} = \frac{(T_1 - T_2)}{L}$$

(*) Newton law of cooling

It is the Fundamental law of heat convection it state that rate of heat transfer is directly proportional to temp difference between the surface and the fluid. and surface area perpendicular to heat flow direction

$Tw > T\infty$

$$\therefore Q \propto A \cdot (Tw - T\infty)$$

$$Q = hA(Tw - T\infty)$$

where h = constant of proportionality called co-efficient of convective heat transfer (W/m^2K)

$$\therefore \quad Q = \frac{(T_w - T_\infty)}{1/hA}$$

$\therefore$ convective thermal resistance (R)

$$R = \frac{T_w - T_\infty}{Q} = \frac{1}{h \cdot A}$$

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② The stefan Boltzmann law of ~~heat~~ Thermal Radiation :

It state that rate of the radition heat transfer per unit area from a black surface is directly proportional to fourth powes of absolute temp.

$$\frac{Q}{A} \propto T_s^4 \qquad \frac{Q}{A} = \sigma T_s^4$$

T_s - absolute temp of surface K

σ - const of proportionality is called steafan Boltzmann const has value $5.67 \times 10^{-8}\ W/m^2K^4$

heat flux emmited by real surface is less than that of black surface and given by

$$\frac{Q}{A} = \sigma \varepsilon (T_s^4)$$

ε = emmisivity

∴ Net rate of radiation heat exchange betn real surface and its surrounding is

$$\frac{Q}{A} = \sigma \varepsilon (T_s^4 - T_\infty^4)$$

T_∞ = surrounding temp

T_s - surface temp

① problem on conduction

1) The wall of furnace is constructed from 15 cm thick fire brick having constant thermal conductivity of 1.7 W/mK. The two side of the wall are maintained at 1400°K and 1150°K resp. what is the rate of heat loss through the wall which is 50 cm × 3 m on a side.

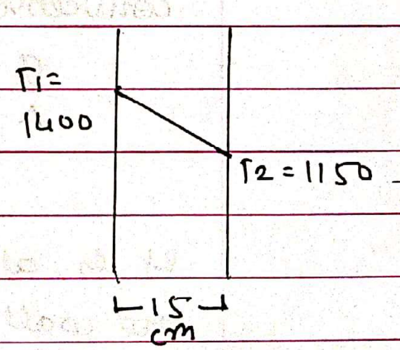
Soln ④ $T_1 = 1400\,K$ $T_2 = 1150$

$A = 50\text{ cm} \times 3\text{ m}$

$= 0.5 \times 3 = 1.5\,m^2$

$K = 1.7\ W/m\,K$

$L = 0.15\ m$



find Heat loss.

$$\therefore\ Q = \frac{KA(T_1 - T_2)}{L}$$

$$= \frac{1.7 \times 1.5 \times (1400 - 1150)}{0.15} = 4250\ W.$$

conduction

8) A refrigerator stands in a room where air temp is 21°C. The surface temp on the outside of refrigerator is 16°C. The sides are 30 mm thick and has an equivalent thermal conductivity of 0.10 w/mK. The heat transfer coefficient on the out side is 10 W/m²K. Assume one dimensional conduction through sides. Calculate the net heat flow rate and inside surface temp of the refrigerator.

Soln

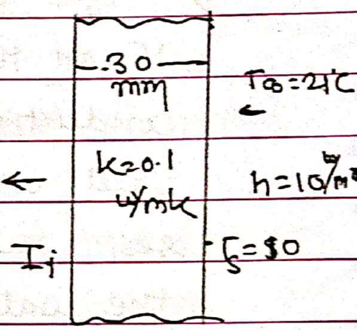
$T_\infty = 21°C$

$T_s = 16°C$

$L = 30\ mm = 0.03\ m$

$k = 0.1\ W/m°K$

$h = 10\ W/m^2K$



convective heat flux

$$\therefore \frac{Q}{A} = h(T_\infty - T_s)$$

$$= 10(21-16) = 50\ W/m^2$$

It is also the net heat conducted through the wall

$$\therefore q = \frac{Q}{A} = \frac{k(T_s - T_i)}{L}$$

$$50 = \frac{(0.1)(16-T_i)}{0.03}$$

$$\boxed{T_i = 1°C}$$

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(4) A black surface is positioned in vacume container so that it absorb solar radiant energy at the rate of 950 w/m². if the surface conducts no heat to surrounding determine its equilibrium temp.

Sol^n : Given - black body aborb solar energy in vaccum = $q = 950\ w/m^2$

$\sigma = 5.67 \times 10^{-8}\ w/m^2 K^4$

$\therefore$ Radient heat flux for black surface

$q = \sigma T_s^4$

$(T_s) = \left(\frac{q}{\sigma}\right)^{1/4} = \left(\frac{950}{5.67 \times 10^{-8}}\right)^{1/4}$

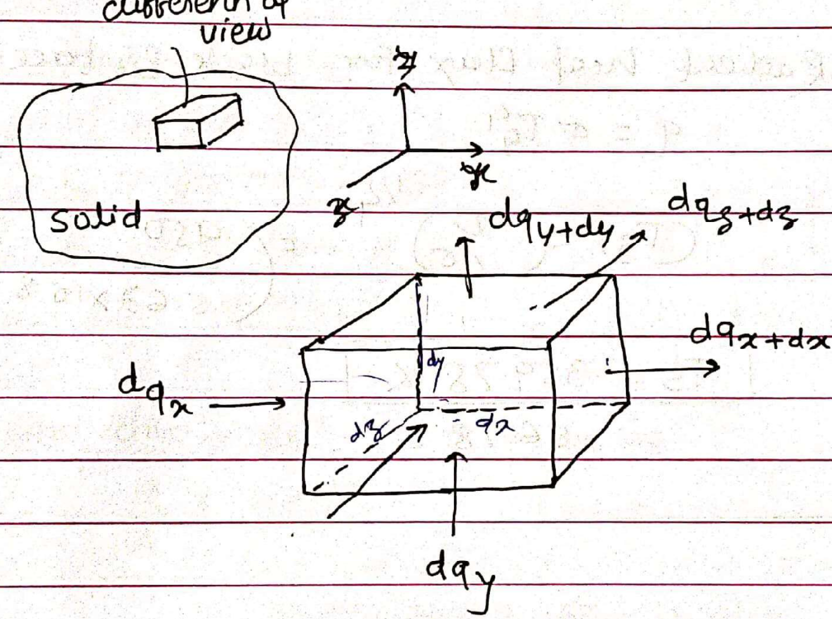
$\boxed{T_s = 359.78\ K}$

$= 86.78°C$

★ Three dimensional equation of heat conduction (In differential equation) ★

cartesian - cordinate

consider the differential element dx dy dz located (aribitrarily) within a solid material. assume that there is heat generation in the material due to some cause (eg. passage of an electric current) at the rate of $\bar{q}$ units.



differential equation of heat conduction

In general, The rate of heat generation may vary from point to point and with time. Thus thus

$$\bar{q} = f(x, y, z\, t)$$

In simpler situation, $\bar{q}$ may vary only with space and not time or may even be a constant

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From Fouries law, we can written down expression for the heat conducted into and out of six faces of differential element. These heat flows are indicated by the symbol dq_x, dq_{x+dx}, dq_{y+dy} + dq_y, dq_z, dq_{z+dz}.

$$dq_x = -k \frac{\partial T}{\partial x} dy dz$$

$$dq_{x+dx} = dq_x + \frac{\partial}{\partial x}(dq_x) dx$$

$$= -\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) dx \right] dy dz.$$

Similarily

$$dq_y = -k\frac{\partial T}{\partial y} dx dz$$

$$dq_{y+dy} = -\left[k\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) dy \right] dx dz.$$

$$dq_z = -k\frac{\partial T}{\partial z} dx dy$$

$$dq_{z+dz} = -\left[k\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) dz \right] dx dy$$

Therefore, the net amount of heat conducted into differential element per unit time

$$= (dq_x + dq_y + dq_z) - (dq_{x+dx} + dq_{y+dy} + dq_{z+dz})$$

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$$= \left[\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)\right] dx\,dy\,dz$$

The quantity of Total heat generated in the differential ~~in the~~ ~~el~~ element per unit time

$$= \bar{q}\, dx\,dy\,dz$$

The rate of change of internal energy of the element

$$= \rho c_p \frac{\partial T}{\partial t}\, dx\,dy\,dz$$

By the first law of thermodynamics the sum of net heat conducted into element the heat generated in it per unit time must be equal to rate of change of internal energy of the element.

$$\therefore \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \bar{q}_g = \rho c_p \frac{\partial T}{\partial t}$$

This is the required differential equation for unsteady state heat conduction for anisotropic material

1) for isotropic material, thermal conductivity is constant. $k = \text{const.}$

$$\therefore k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \bar{q}_g = \rho c_p \frac{\partial T}{\partial t}$$

$$\nabla^2 T + \frac{\bar{q}}{k} = \frac{\rho c_p}{k} \frac{\partial T}{\partial t}$$

$$\nabla^2 T + \frac{\bar{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\therefore \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$ called Laplace operator

$\alpha = \frac{k}{\rho c_p}$ is called thermal diffusivity (m^2/s)

2) if there is no internal heat generation within the material (i.e. $\bar{q} = 0$) the governing equation reduces to fourier equation as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

3) for steady state condition $\frac{\partial T}{\partial t} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2} + \frac{\bar{q}}{k} = 0$$

The three dimensional differential equation for steady state heat conduction with const thermal conductivity called poission eq.

4) If the solid has no heat generation

$\bar{q} = 0$

$$\therefore \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

$$\nabla^2 T = 0$$

above equation is the three dimensional differential equation for steady state heat conduction without heat generation. with constant thermal conductivity.
also called
Laplace equation.

★ Thermal conductivity

Thermal conductivity is the property of material and is define as the ability of materials to conduct the heat through it. It can also be interpreted as the rate of heat transfer through a unit thickness of materal per unit area per unit temp difference.
unit : w/m°k or w/m°c
Highest in solid, lowest in gases.

The value of thermal conductivity depends upon the manner in which energy is transferred. pure metals allows faster transmission of heat energy through vibration of crystal lattice. Therefore the metal in pure state has maximum thermal conductivity and is a good conductor.

The thermal conductivity decreases in ~~Increases~~ the impurities in a metals. most non-metals are poor-conductor of heat transfer therefore have low value of thermal conductivity are called thermal insulators.

In gases, The faster the molecule move, the faster, they will transport energy. Therefore, The thermal conductivity of gases depends on the square root of absolute temperature ~~The thermal conductivity of.~~

The physical Mechanism of heat conduction in liquid is also same as in gases, however, the mechanism is slightly complex due to close spacing of molecules and molecular attraction.

Effect of thermal conductivity by temperature

The temp is measure of kinetic energy of molecule of a substance. Thus thermal conductivity is a function of temperature. it also change with pressure in fluid.

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* Solid

1) Metals: The heat may be conducted in metals by two mechanism ⓐ migration of free electrons & b) lattice vibration. In general the presence of the electron gas (large free electrons) in metals, make it good conductor of heat, but the conduction also takes place due to vibrational energy in lattice structure. The flow of free electrons in metals result in increase in value of thermal conductivity several times. But at the same time, due to increase in temp, vibration of the molecules in the metal become voilent and they obstruct the flow of free electrons and contribution to the heat conduction by free electron decreases. Thus it may results in net decrease in the heat flow.

Hence, for most of metals, the value of thermal conductivity decreases as temp increases.

Thermal conductivity of mercury increases with increase in temp. (exceptional case)

2) Non-metals :- due to absence of free electron in non-metals, the heat condution is only due to lattice vibration. As temp increases, hence, the rate of heat flow increases in non-metals. Thus the thermal conductivity

increase with increase in temp.

liquid: for most of the liquid, the thermal conductivity decreases with increase in the temp. But water and glycerine are the exceptional case. the thermal conductivity of liquid is independent of pressure. as general rule thermal conductivity of liquid decreases with increase in molecular weight. ~~the value of thermal conductivity of liquid are taken~~

Gases: For the gases, the molecule are in continuous random motion, as a temp increases, velocities of the molecules become higher than in some lower temp region. The molecules move from high temp region to low temp region and give up its energy through collision to lower molecules. Thus the thermal conductivity of gases increase with increase in temp. and it is proportional to square root of the ~~molecule~~ absolute temp. it is also affected by change in pressure and humidity.

Thermal diffusivity (α)

It is the important characteristic quantity for the unsteady conduction situations.
It is the ratio of thermal conductivity K of the medium to heat capasity ρC (α) (m^2/s)

$$\therefore \quad \alpha = \frac{K}{\rho C} \quad \frac{m^2}{s}$$

Theremal conductivity 'k' represent how well the material can conduct heat and heat capasity ρC represents how much energy a material can store per unit volume. therefore the the thermal diffusivity of material viewed as the ratio of heat conducted through the material to the heat stored per unit volume. In other words the thermal diffusivity of material is associated with propagation of heat energy into the medium during change of temp with time. higher the thermal diffusivity, faster the propagation of heat into the medium & its temp will change with time.

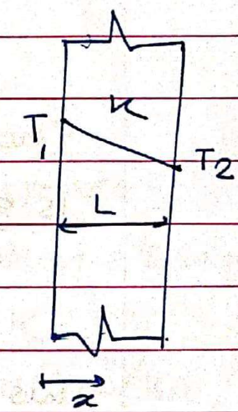
one dimensional steady state conduction without heat Generation.

1) Plane wall

consider a plane wall of homogenous material through which heat is flowing only in x- direction L- thickness. it's left face at $x=0$ is at temperature T_1 & right face temp T_2 at $x=L$. The wall has

The general heat conduction equation in cartersian coordinate is given by

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\bar{q}}{k} = \frac{1}{\alpha}\frac{dT}{dt}$$



∴ as their is no internal heat generation. $\bar{q}=0$

∴ The heat conduction takes place under steady state condition $\frac{dT}{dt}=0$

one dimensional - $\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2} = 0$

$$\frac{\partial^2 T}{\partial x^2} = 0 \quad -(I) \quad \text{or} \quad \frac{d^2 T}{dx^2} = 0$$

integrating above equation $\frac{\partial T(x)}{dx} = C_1$ —(II)

again integrating

$$T(x) = C_1 x + C_2 \quad -(III)$$

where C_1 and C_2 constant of integration and are

evaluated with use of boundary conditions.
The boundary conditions

$$T(x) = T_1 \quad \text{at } x = 0$$
$$T(x)\ T_2 \quad \text{at } x = L$$

put in (III) using first boundary condition

$$C_2 = T_1$$

put in (III) using second boundary conditions

$$T_2 = C_1 L + T_1$$

$$C_1 = \frac{T_2 - T_1}{L}$$

Then the temp distribution in plane wall is given by

$$T(x) = (T_2 - T_1)\frac{x}{L} + T_1 \quad \text{— (IV)}$$

This is the temp distribution $T(x)$ in plane wall. It is the linear function of x as shown in fig.

differentiate eq. (IV)

$$\text{diff} \quad \frac{dT(x)}{dx} = \frac{T_2 - T_1}{L}$$

∴ Heat flux from fourier law

$$q(x) = -k\frac{dT(x)}{dx} = -k\frac{(T_2 - T_1)}{L}$$
$$= k\frac{(T_1 - T_2)}{L}$$

∴ Total Heat flow rate Q through area 'A' normal to direction

$$\boxed{Q = KA\left(\frac{T_1 - T_2}{L}\right)}$$