UNIT-2

CO1: To understand the basic concept of mode of heat transfer.

CO2: To apply non-dimensional numbers to evaluate and validate heat transfer parameters.

CO-3: To analyze the complex problems of heat transfer with proper boundary conditions.

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2	Unsteady state heat conduction for slab, cylinder and sphere, Heisler chart.	2
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Heat transfer from extended surfaces

Heat transfer between a solid surface and a moving fluid is governed by the Newton's cooling law:

$$q = hA(T_s - T_\infty)$$

where T_s is the surface temperature and T_∞ is the fluid temperature. Therefore, to increase the convective heat transfer, one can

Increase the temperature difference $(T_s - T_\infty)$ between the surface and the fluid.

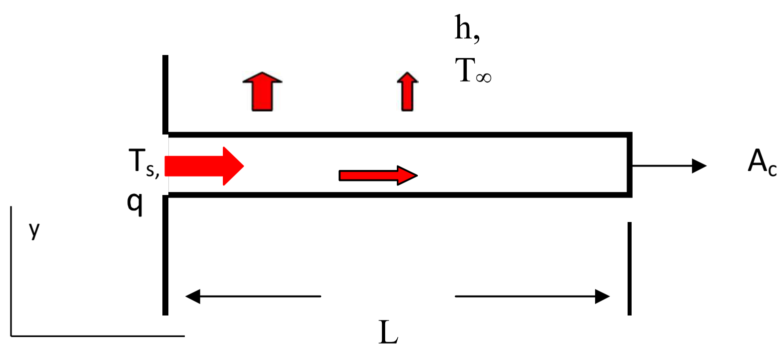
Increase the convection coefficient h. This can be accomplished by increasing the fluid flow over the surface since h is a function of the flow velocity and the higher the velocity, the higher the *h*. Example: a cooling fan.

Increase the contact surface area A. Example: a heat sink with fins.

Many times, when the first option is not in our control and the second option (i.e. increasing *h*) is already stretched to its limit, we are left with the only alternative of increasing the effective surface area by using fins or extended surfaces. Fins are protrusions from the base surface into the cooling fluid, so that the extra surface of the protrusions is also in contact with the fluid. Most of you have encountered cooling fins on air-cooled engines (motorcycles, portable generators, etc.), electronic equipment (CPUs), automobile radiators, air conditioning equipment (condensers).

In the study of heat transfer, fins are surfaces that extend from an object to increase the rate of heat transfer to or from the environment by increasing convection. The amount of conduction, convection, or radiation of an object determines the amount of heat it transfers. Increasing the temperature gradient between the object and the environment, increasing the convection heat transfer coefficient, or increasing the surface area of the object increases the heat transfer. Sometimes it is not feasible or economical to change the first two options. Thus, adding a fin to an object, increases the surface area and can sometimes be an economical solution to heat transfer problems.

Consider the cooling fin shown below:

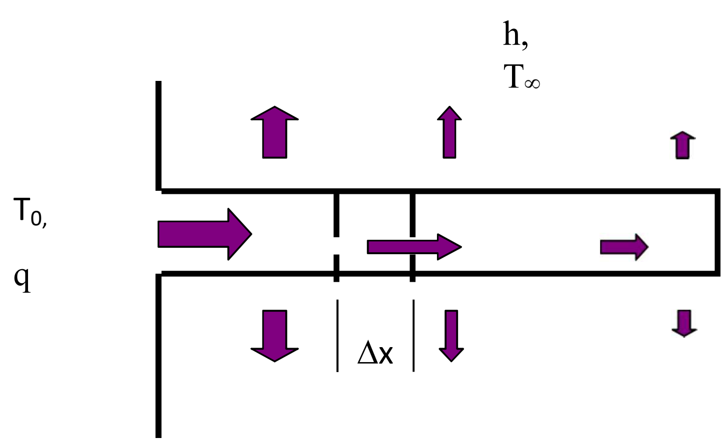


The fin is situated on the surface of a hot surface at T_s and surrounded by a coolant at temperature T_∞, which cools with convective coefficient, h. The fin has a cross sectional area, A_c, (This is the area through with heat is conducted.) and an overall length, L.

Note that as energy is conducted down the length of the fin, some portion is lost, by convection, from the sides. Thus the heat flow varies along the length of the fin.

We further note that the arrows indicating the direction of heat flow point in both the x and y directions. This is an indication that this is truly a two- or three-dimensional heat flow, depending on the geometry of the fin. However, quite often, it is convenient to analyse a fin by examining an equivalent one–dimensional system. The equivalent system will involve the introduction of heat sinks (negative heat sources), which remove an amount of energy equivalent to what would be lost through the sides by convection.

Consider a differential length of the fin.



Across this segment the heat loss will be $h\cdot(P\cdot\Delta x)\cdot(T-T_\infty)$, where P is the perimeter around the fin.

Equating the heat source to the convective loss:

$$\& \frac{-h \cdot P \cdot \left(T - T_{\infty}\right)}{A_c}$$

Substitute this value into the General Conduction Equation as simplified for One-Dimension, Steady State Conduction with Sources, where m is

$$m^2 = \frac{h \cdot P}{k \cdot A_c}$$

then

$$dx^2 - m^2 \cdot \theta = 0$$

This equation is a Second Order, Homogeneous Differential Equation.

Solution of the Fin Equation

We apply a standard technique for solving a second order homogeneous linear differential equation.

Try $\theta = e^{\alpha \cdot x}$. Differentiate this expression twice:

$$\frac{d\theta}{dx} = \alpha \cdot e^{\alpha \cdot x}$$

$$\frac{d^2\theta}{dx^2} = \alpha^2 \cdot e^{\alpha x}$$

Substitute this trial solution into the differential equation:

$$\alpha^2 \cdot e^{\alpha \cdot x} - m^2 \cdot e^{\alpha \cdot x} = 0$$

Equation provides the following relation:

$$\alpha = \pm\ m$$

We now have two solutions to the equation. The general solution to the above differential equation will be a linear combination of each of the independent solutions.

Then:

$$\theta = A \cdot e^{m \cdot x} + B \cdot e^{-m \cdot x}$$

where A and B are arbitrary constants which need to be determined from the boundary conditions. Note that it is a 2nd order differential equation, and hence we need two boundary conditions to determine the two constants of integration.

An alternative solution can be obtained as follows: Note that the hyperbolic sin, sinh, the hyperbolic cosine, cosh, are defined as:

$$\sinh(m \cdot x) = \frac{e^{m \cdot x} - e^{-m \cdot x}}{2} \qquad \cosh(m \cdot x) = \frac{e^{m \cdot x} + e^{-m \cdot x}}{2}$$

We may write:

$$C \cdot \cosh(m \cdot x) + D \cdot \sinh(m \cdot x) = C \cdot \frac{e^{m \cdot x} + e^{-m \div x}}{2} + D \cdot \frac{e^{m \cdot x} - e^{-m \div x}}{2} = \frac{C + D}{2} \cdot e^{m \cdot x} + \frac{C - D}{2} \cdot e^{-m \cdot x}$$

We see that if (C+D)/2 replaces A and (C-D)/2 replaces B then the two solutions are equivalent.

$$\theta = C \cdot \cosh(m \cdot x) + D \cdot \sinh(m \cdot x)$$

Generally the exponential solution is used for very long fins, the hyperbolic solutions for other cases.

Boundary Conditions:

Since the solution results in 2 constants of integration we require 2 boundary conditions. The first one is obvious, as one end of the fin will be attached to a hot surface and will come into thermal equilibrium with that surface. Hence, at the fin base,

$$\theta(0) = T_0 - T_\infty \equiv \theta_0$$

The second boundary condition depends on the condition imposed at the other end of the fin. There are various possibilities, as described below.

Very long fins:

For very long fins, the end located a long distance from the heat source will approach the temperature of the surroundings. Hence,

$$\theta(\infty) = 0$$

Substitute the second condition into the exponential solution of the fin equation:

$$\theta(\infty) = 0 = A\cdot \overbrace{e^{m\cdot\infty}}^{\infty} + B\cdot \overbrace{e^{-m\cdot\infty}}^{0}$$

The first exponential term is infinite and the second is equal to zero. The only way that this equation can be valid is if A = 0. Now apply the second boundary condition.

$$\theta(0) = \theta_0 = B\cdot e^{-m\cdot 0} \Rightarrow B = \theta_0$$

The general temperature profile for a very long fin is then:

$$\theta(x) = \theta_0 \cdot e^{-m\cdot x}$$

If we wish to find the heat flow through the fin, we may apply Fourier Law:

$$q = -k\cdot A_c \cdot \frac{dT}{dx} = -k\cdot A_c \cdot \frac{d\theta}{dx}$$

Differentiate the temperature profile:

$$\frac{d\theta}{dx} = -\theta_o \cdot m\cdot e^{-m\cdot x}$$

The insulated tip fin

Assume that the tip is insulated and hence there is no heat transfer:

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

The solution to the fin equation is known to be:

$$\theta = C\cdot\cosh(m\cdot x) + D\cdot\sinh(m\cdot x)$$

Differentiate this expression.

$$\frac{d\theta}{dx} = C\cdot m\cdot\sinh(m\cdot x) + D\cdot m\cdot\cosh(m\cdot x)$$

Apply the first boundary condition at the base:

$$\theta(0) = \theta_0 = C\overset{0}{\sinh(m\cdot 0)} + D\overset{1}{\cosh(m\cdot 0)}$$

So that D = θ_0. Now apply the second boundary condition at the tip to find the value of C:

$$\frac{d\theta}{dx}(L) = 0 = Cm\sinh(m\cdot L) + \theta_0 m\cosh(m\cdot L)$$

which requires that

$$C = -\theta_0\frac{\cosh(mL)}{\sinh(mL)}$$

This leads to the general temperature profile:

$$\theta(x) = \theta_0\frac{\cosh m(L-x)}{\cosh(mL)}$$

We may find the heat flow at any value of x by differentiating the temperature profile and substituting it into the Fourier Law:

$$q = -k \cdot A_c \cdot \frac{dT}{dx} = -k \cdot A_c \cdot \frac{d\theta}{dx}$$

So that the energy flowing through the base of the fin is:

$$q = \sqrt{hPkA_c}\,\theta_0 \tanh(mL) = M\theta_0 \tanh(mL)$$

If we compare this result with that for the very long fin, we see that the primary difference in form is in the hyperbolic tangent term. That term, which always results in a number equal to or less than one, represents the reduced heat loss due to the shortening of the fin.

Other tip conditions:

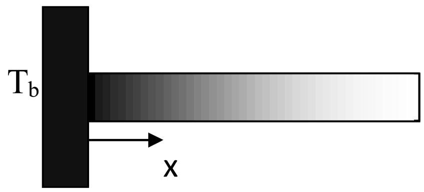
We have already seen two tip conditions, one being the long fin and the other being the insulated tip. Two other possibilities are usually considered for fin analysis: (i) a tip subjected to convective heat transfer, and (ii) a tip with a prescribed temperature. The expressions for temperature distribution and fin heat transfer for all the four cases are summarized in the table below.

Case	Tip Condition	Temp. Distribution	Fin heat transfer
A	Convection heat transfer: $h\theta(L)=-k(d\theta/dx)_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\theta_o \dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic $(d\theta/dx)_{x=L}=0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M\theta_0 \tanh mL$
C	Given temperature: $\theta(L)=\theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh m(L-x) + \sinh m(L-x)}{\sinh mL}$	$M\theta_0 \dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$
D	Infinitely long fin $\theta(L)=0$	e^{-mx}	$M\,\theta_0$

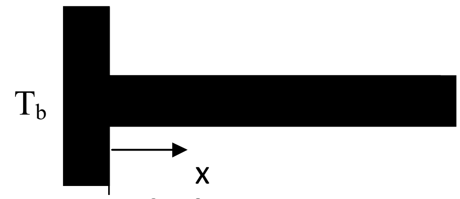
Fin Efficiency

The fin efficiency is defined as the ratio of the energy transferred through a real fin to that transferred through an ideal fin. An ideal fin is thought to be one made of a perfect or infinite conductor material. A perfect conductor has an infinite thermal conductivity so that the entire fin is at the base material temperature.

$$\eta = \frac{q_{real}}{q_{ideal}} = \frac{\sqrt{h\cdot P\cdot k\cdot A_c}\cdot\theta_L\cdot\tanh(m\cdot L)}{h\cdot(P\cdot L)\cdot\theta_L}$$



Real situation



Ideal situation

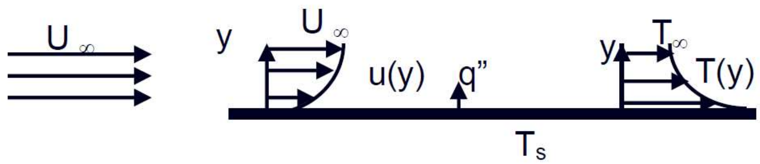
CONVECTION:

Main purpose of convective heat transfer analysis is to determine:

Flow field, temperature field in fluid ,heat transfer coefficient, h

Heat transfer through a solid is always by conduction, since the molecules of a solid remain at relatively fixed positions. Heat transfer through a liquid or gas, however, can be by conduction or convection, depending on the presence of any bulk fluid motion. Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it. Therefore, Conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid

$$q''_{conv} = -k_f \left.\frac{\partial T}{\partial y}\right)_{y=0} = h(T_s - T_\infty)$$

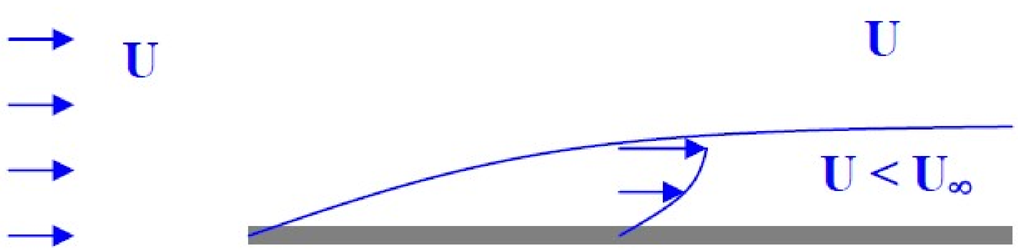


Convection classified in two types

i) Natural convection
ii) Forced convection

FORCED CONVECTION:

An internal flow is surrounded by solid boundaries that can restrict the development of its boundary layer, for example, a pipe flow. An external flow, on the other hand, are flows over bodies immersed in an unbounded fluid so that the flow boundary layer can grow freely in one direction. Examples include the flows over airfoils, ship hulls, turbine blades.



Fluid particle adjacent to the solid surface is at rest
• These particles act to retard the motion of adjoining layers
• boundary layer effect

Inside the boundary layer, we can apply the following conservation principles:
Momentum balance: inertia forces, pressure gradient, viscous forces, body forces
Energy balance: convective flux, diffusive flux, heat generation, energy storage

FORCED CONVECTION CORRELATIONS:

The heat transfer coefficient is a direct function of the temperature gradient next to the wall,

the physical variables on which it depends can be expressed as follows:
h=f(fluid properties, velocity field ,geometry, temperature etc.)

As the function is dependent on several parameters, the heat transfer coefficient is usually expressed in terms of **correlations involving pertinent non-dimensional numbers**.

Forced convection: **Non-dimensional groupings**

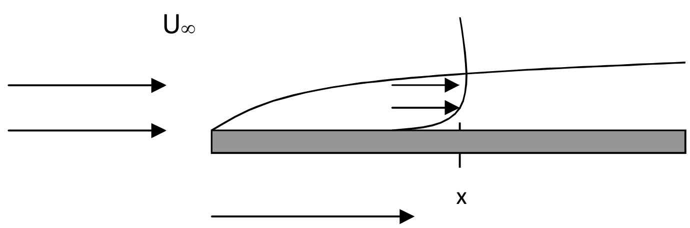
- **Nusselt No**. Nu = hx / k = (convection heat transfer strength)/ (conduction heat transfer strength)
- **Prandtl No.** Pr = ν/α = (momentum diffusivity)/ (thermal diffusivity)
- **Reynolds No**. Re = U x / ν = (inertia force)/(viscous force)

Viscous force provides the dampening effect for disturbances in the fluid. If dampening is strong enough ⇒ **laminar flow**
Otherwise, instability ⇒ **turbulent flow** ⇒ **critical Reynolds number**

For forced convection, the heat transfer correlation can be expressed as

Nu=f (Re, Pr)

The convective correlation for laminar flow across a flat plate heated to a constant wall temperature is:



$$Nu_x = 0.323 \cdot Re_x^{1/2} \cdot Pr^{1/3}$$

where

$$Nu_x \equiv h \cdot x/k$$

$$Re_x \equiv (U_\infty \cdot x \cdot \rho)/\mu$$

$$Pr \equiv c_P \cdot \mu/k$$

PHYSICAL MEANING OF PRANDTL NUMBER

The Prandtl number was introduced earlier.

If we multiply and divide the equation by the fluid density, ρ, we obtain:

$$Pr \equiv (\mu/\rho)/(k/\rho \cdot c_P) = \upsilon/\alpha$$

The Prandtl number may be seen to be a ratio reflecting the ratio of the rate that viscous forces penetrate the material to the rate that thermal energy penetrates the material. As a consequence the Prandtl number is proportional to the rate of growth of the two boundary layers:

$$\delta/\delta_t = Pr^{1/3}$$

PHYSICAL MEANING OF NUSSELT NUMBER

The Nusselt number may be physically described as well.

$$Nu_x \equiv h \cdot x/k$$

If we recall that the thickness of the boundary layer at any point along the surface, δ, is also a function of x then

$$Nu_x \propto h \cdot \delta/k \propto (\delta/k \cdot A)/(1/h \cdot A)$$

We see that the Nusselt may be viewed as the ratio of the conduction resistance of a material to the convection resistance of the same material.

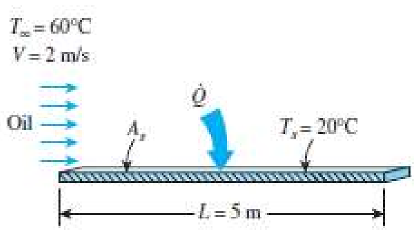
Students, recalling the Biot number, may wish to compare the two so that they may distinguish the two.

$$Nu_x \equiv h \cdot x/k_{fluid} \qquad Bi_x \equiv h \cdot x/k_{solid}$$

The denominator of the Nusselt number involves the thermal conductivity of the **fluid** at the solid-fluid convective interface; the denominator of the Biot number involves the thermal conductivity of the **solid** at the solid-fluid convective interface.

NUMERICAL:

1. Engine oil at 60^0C flows over the upper surface of a 5-m-long flat plate whose temperature is 20^0C with a velocity of 2 m/s in following figure. Determine the total drag force and the rate of heat transfer per unit width of the entire plate.



Solution: Engine oil flows over a flat plate. The total drag force and the rate of heat transfer per unit width of the plate are to be determined

1 The flow is steady and incompressible. **2** The critical Reynolds number is Re = 5 × 10^5

The properties of engine oil at the film temperature of $T_f = (T_s + T_\infty)/2 = (20+60)/2 = 40^0$C are ρ =876 kg/m^3 , Pr= 2962,*k*=0.1444 W/m.K ,*v* =2.485 10 $^{-4}$ m^2/s

Noting that *L* =5 m,

The Reynolds number at the end of the plate is

$$Re_L = VL/v$$
$$= (2 \times 5) / 2.485 \times 10^{-4}$$
$$= 4.024 \times 10^4$$

Which is less than the critical Reynolds number. Thus we have laminar flow over the entire plate, and the average friction coefficient is

C_f =1.33 $Re_L{}^{20.5}$
=1.33 3 $(4.024 \times 10^4)^{20.5}$
= 0.00663

Noting that the pressure drag is zero and thus C_D=C_f for parallel flow over a flat plate, the drag force acting on the plate per unit width becomes

$F_D = C_f A(\rho V^2/2)$
$= \frac{0.00663(5\times1\ m^2)\ (876\ kg/m^3)(2\ m/s)^2}{2}$
=**58.1 N**

The total drag force acting on the entire plate can be determined by multiplying the value obtained above by the width of the plate.

The Nusselt number is determined using the laminar flow relations
for a flat plate,

Nu=hL/k
=0.664 $Re_L{}^{0.5}\ Pr^{1/3}$
=0.664 3 $(4.024 \times 10^4)^{0.5} \times 2962^{1/3}$
=1913

Then,h =(kNu)/(LK)= 50.1444 W/m·K
=55.25 $W/m^2 \cdot K$

And $Q = hAs(T^{`} - Ts)$
=(55.25)(5)(60-20)^{0}C
=**11,050 W**